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SECTION 01

What Is This Tool?

The **Markowitz Frontier** is a two-asset portfolio optimiser built on Harry Markowitz's Nobel Prize-winning Modern Portfolio Theory (1952). It answers one of the most fundamental questions in finance: given two investments, how much of each should I hold to get the best possible return for the risk I am willing to take?

The tool fetches three years of real price history from Yahoo Finance, computes each asset's historical return and volatility, calculates how correlated they are, and then maps every mathematically possible combination of the two into a single visual chart — the **Efficient Frontier**. The optimal portfolio is identified automatically.

Who should use this tool?

Finance students completing portfolio-theory assignments, investment analysts evaluating a two-asset allocation, fund managers studying diversification between two securities, and any professional who needs a rigorous, visual answer to "how do I split my money between these two assets?"

What the tool does NOT do

This tool optimises a **two-asset portfolio only**. Real-world portfolios typically hold many assets. The two-asset model is the purest demonstration of the diversification principle and the efficient frontier concept, and its output is directly applicable to any sleeve, pair trade, or benchmark vs. active allocation decision.

SECTION 02

Quick Start

You can run a complete optimisation in under thirty seconds.

1 ENTER TWO TICKER SYMBOLS

Type the Yahoo Finance ticker for Asset A and Asset B — for example, RELIANCE.NS and HDFCBANK.NS for Indian stocks, or AAPL and MSFT for US stocks. International tickers use a suffix: .NS for NSE, .BO for BSE, .L for London, .HK for Hong Kong.

2 CLICK "PLOT FRONTIER"

The tool fetches three years of monthly price data from Yahoo Finance, calculates each asset's annualised return and volatility, computes the correlation between them, and builds the efficient frontier. All automatically — no spreadsheets required.

3 READ THE CHART AND THE SUMMARY STRIP

The coloured dot cloud shows all possible portfolios. The ★ star marks the optimal (maximum Sharpe) portfolio. The ◆ diamond marks the minimum-risk portfolio. The dashed line shows the Capital Market Line. The stats strip below the chart button shows the exact weights and metrics.

4 EXPORT OR SAVE (TRIAL / PREMIUM)

Click Export PDF to download a formatted report showing all parameters, the frontier chart, and the optimal portfolio weights. Use Save Model to store the inputs for future sessions.

SECTION 03

Understanding the Inputs

All financial parameters — return, volatility, and correlation — are filled in automatically when you click Plot Frontier. You only need to enter the two ticker symbols. The fields are described below so you understand what each number means.

TICKER A & B

The Yahoo Finance symbol for each asset. **US stocks:** AAPL, MSFT, SPY, QQQ. **Indian stocks (NSE):** RELIANCE.NS, INFY.NS, TCS.NS. **Indian stocks (BSE):** RELIANCE.BO, HDFCBANK.BO. **Crypto:** BTC-USD, ETH-USD. **Indices:** ^GSPC (S&P 500), ^NSEI (Nifty 50), ^BSESN (Sensex). Up to 20 characters. Must be a valid, tradeable symbol on Yahoo Finance.

ANN. RETURN %

The annualised historical return for the asset over the last three years, expressed as a percentage. Calculated as the average of 36 monthly returns, multiplied by 12. This is the arithmetic annualised return — it reflects the average pace of growth, not the compounded growth rate.

```
Ann. Return = Avg Monthly Return × 12
```

Populated automatically. A value of 15 means the asset returned roughly 15% per year on average over the last three years.

ANN. VOLATILITY %

The annualised standard deviation of monthly returns, expressed as a percentage. It measures how much the asset's price fluctuates. Higher volatility = higher risk. A stock with 30% annual volatility has roughly a two-thirds chance of ending any given year within ±30% of where it

started.

```
Ann. Volatility = Monthly Std Dev × √12
```

Populated automatically. Typical ranges: large-cap equities 15-25%, small-caps 25-45%, crypto 60-100%.

CORRELATION P

Measures how the two assets move relative to each other. Ranges from −1 to +1. +1 = they always move in the same direction (no diversification benefit). 0 = completely independent movements. −1 = they always move in opposite directions (perfect hedge). In practice, most equity pairs have correlations between 0.3 and 0.7.

Calculated automatically from the overlapping monthly return history of both tickers. You can override it manually using the slider.

RANDOM PORTFOLIOS

The number of random weight combinations to simulate and plot as coloured dots. More portfolios = a denser, smoother cloud that better visualises the full opportunity set. 500 is good for quick analysis; 2000–5000 is better for presentations or PDFs. Range: 100 – 5000.

RISK-FREE RATE %

The return available on a zero-risk investment — typically a short-term government bond yield. In India this is approximately the 91-day T-Bill rate (~6.5–7% as of 2025). In the US it is the Fed Funds rate or 3-month Treasury yield (~5%). This rate is the starting point of the Capital Market Line and the denominator anchor of the Sharpe Ratio. Default: 4.5%. Adjust to match the prevailing rate in your market.

> **Project Info fields** (Title, Name, Roll, Date) are optional but populate the PDF report header and the chart's top-right metadata. Fill them in if you are submitting a course assignment or client report.

SECTION 04

Reading the Chart

The chart plots **Volatility (Risk) on the x-axis** and **Expected Return on the y-axis**. Every point on the chart represents a specific portfolio — a specific split between Asset A and Asset B. Moving left on the x-axis means lower risk; moving up on the y-axis means higher return. The goal is to be as far up and left as possible.

The Dot Cloud — The Opportunity Set

Each dot is one simulated portfolio with a specific weight split between the two assets (e.g. 40% in A, 60% in B). The colour of each dot encodes its **Sharpe Ratio**: blue = low Sharpe (poor risk-adjusted return), green = medium, red/orange = high Sharpe (excellent risk-adjusted return). The full cloud shows every reachable risk-return combination. Portfolios inside the cloud are always dominated — for the same risk, there is a better-returning option on the boundary.

The Curved Black Line — The Efficient Frontier

This curve is the outer boundary of the opportunity set. Every portfolio ON this curve is **efficient**: for its level of risk, it delivers the highest possible return. No portfolio can exist above or to the left of this curve. Portfolios below or to the right of the curve are

exist above or to the left of this curve. Portfolios below or to the right of the curve are inefficient — a rational investor should never hold them. The curve bows leftward because of the diversification effect: when two imperfectly correlated assets are combined, the portfolio risk is less than the weighted average of the individual risks.

The Dashed Teal Line — The Capital Market Line (CML)

The Capital Market Line is a **straight line** that starts at the risk-free rate on the y-axis (zero volatility) and runs through the Maximum Sharpe portfolio (★), then continues beyond it. It represents the best risk-return combinations available when an investor can also hold cash (or borrow). Any point on the CML is achievable by:

- **Left of ★:** Invest part in the Max Sharpe portfolio, keep the rest as cash (risk-free). Lower risk, lower return.
- **At ★:** 100% in the Max Sharpe portfolio. Pure risky optimal.
- **Right of ★:** Borrow at the risk-free rate to invest more than 100% in the Max Sharpe portfolio (leverage). Higher risk, higher return.

The CML is always above or tangent to the efficient frontier. Its slope is the Sharpe Ratio of the Max Sharpe portfolio — the highest achievable risk-adjusted return.

The Star ★ — Maximum Sharpe / Tangency / Market Portfolio

This single point is the most important result of the entire optimisation. It is simultaneously called:

- **Maximum Sharpe Portfolio** — it has the highest ratio of excess return to risk of any portfolio in the opportunity set.
- **Tangency Portfolio** — it is the exact point where the Capital Market Line touches (is tangent to) the Efficient Frontier. No other point on the frontier has a CML that reaches higher.
- **Market Portfolio** — in Markowitz theory, every rational investor should hold exactly this portfolio of risky assets (combined with varying amounts of cash depending on their personal risk tolerance). The market portfolio is the same for everyone.

The stats strip below the chart button shows the exact weights: e.g., "Weight A: 62.3% · Weight B: 37.7%" and the Sharpe Ratio.

The Diamond ◆ — Minimum Variance Portfolio (MVP)

The leftmost point on the efficient frontier. This portfolio has the **lowest achievable risk** of any combination of the two assets. It minimises volatility regardless of return. It is the ideal starting point for extremely risk-averse investors — even if its return is lower than the Max Sharpe portfolio. The exact weights are shown in the stats strip and the info banner below the chart.

SHARPE BAR

The Colour Bar (Right Side) — Sharpe Ratio Scale

The vertical colour gradient on the right side of the chart acts as a legend for the dot colours. The number at the top is the maximum Sharpe Ratio in the cloud; the number at the bottom is the minimum. Use this to judge whether the overall risk-adjusted return profile of the two-asset universe is attractive.

The Info Banner below the chart

After plotting, a teal-and-gold banner summarises the key numbers: the Max Sharpe portfolio (weights in A and B, return, volatility, Sharpe ratio) and the Minimum Variance portfolio (same metrics). This is the take-away you would put in a presentation or report.

SECTION 05

Core Concepts Explained

The following concepts underpin everything the tool calculates. Understanding them turns the chart from a picture into an actionable decision framework.

Diversification

Combining two assets with a correlation below +1 reduces portfolio risk below the simple weighted average of the individual risks. The lower the correlation, the greater the diversification benefit. When ρ = 0 or negative, the frontier bows far to the left, meaning you can achieve substantially lower risk by holding both assets. This is the "free lunch" of finance.

Risk–Return Trade-off

Higher expected return almost always requires accepting higher risk. The efficient frontier shows this trade-off visually. Portfolios above the Minimum Variance point on the frontier offer progressively higher return in exchange for progressively higher risk. Portfolios below the MVP on the frontier are inefficient — more risk for less return — and should never be chosen.

Mean–Variance Optimisation

The mathematical process of finding portfolios that maximise expected return for a given level of variance (or minimise variance for a given return). Developed by Harry Markowitz in 1952 and the foundation of all modern quantitative portfolio management. The tool solves this problem numerically using a fine grid and Monte Carlo simulation to map the entire feasible set.

The Sharpe Ratio

Measures how much excess return (above the risk-free rate) you receive per unit of volatility. A Sharpe Ratio of 1.0 means you earn 1% of return above the risk-free rate for every 1% of risk taken. Above 1.0 is considered good; above 2.0 is exceptional in equity markets. The tangency portfolio maximises this ratio.

```
Sharpe = (Return - Rf) / Volatility
```

Two-Fund Separation Theorem

All rational investors should hold the same optimal risky portfolio (the Max Sharpe portfolio) and vary their allocation to the risk-free asset based on personal risk tolerance. A conservative investor holds 40% Max Sharpe + 60% cash. An aggressive investor holds 100%+ Max Sharpe using leverage. This is why the CML — not the frontier — represents the true investment opportunity.

Simulation (Monte Carlo)

Rather than evaluating every possible weight analytically, the tool generates hundreds or thousands of random portfolio weights, computes the return and risk of each, and plots them. This Monte Carlo approach paints the full "cloud" of reachable portfolios and makes the efficient frontier intuitively visible. More simulations produce a denser, more accurate picture.

The Tangency Portfolio = Market Portfolio = Max Sharpe: Are they the same thing?

Yes, in the two-asset Markowitz framework, all three names refer to the same portfolio. **Tangency** is the geometric description (where CML touches the frontier). **Max Sharpe** is the optimisation objective (highest risk-adjusted return). **Market Portfolio** is the theoretical interpretation

(every rational investor should hold it). In practice, the market portfolio of an entire economy is approximated by a broad index fund (Nifty 50 ETF, S&P 500 ETF). In this tool, it is the specific combination of your two chosen assets that maximises the Sharpe Ratio.

SECTION 06

Business Applications

The two-asset efficient frontier is directly applicable to a wide range of real-world investment and management decisions:

- **Equity pair allocation.** A fund manager choosing between two stocks in the same sector (e.g., TCS vs Infosys) can use the frontier to find the split that maximises risk-adjusted return, rather than relying on intuition or equal weighting.
- **Asset class diversification.** Analysing a domestic equity (e.g., Nifty ETF) against an alternative asset (gold, REITs, international index) quantifies the diversification benefit and the optimal allocation between the two sleeves.
- **Benchmark vs. active overlay.** A portfolio manager comparing a passive ETF to an active fund can model the frontier to determine the optimal blend that improves Sharpe Ratio versus holding either alone.
- **Hedging analysis.** Pairing a long equity position with an inverse or low-correlation asset (e.g., gold, currency, bond) demonstrates to clients how much risk reduction is achieved and at what return cost. The Minimum Variance Portfolio shows the most effective hedge ratio.
- **Client communication.** The visual frontier is one of the most powerful ways to explain diversification to a non-technical client. The "free lunch" — lower risk for the same return — is immediately visible in the chart. Export the PDF for presentations.
- **Academic and coursework submissions.** The tool provides publication-quality outputs (efficient frontier, CML, Max Sharpe weights, Sharpe Ratio, Min Variance weights) with a branded PDF export, suitable for MBA project submissions and CFA-style case studies.
- **Sensitivity analysis.** Adjust the correlation slider manually after plotting to see how the frontier changes as the relationship between two assets weakens or strengthens over time — useful for stress-testing portfolio assumptions.
- **International diversification.** Compare a domestic index (^NSEI) with a US or global index (^GSPC, IEMG) to demonstrate the benefit of geographic diversification to clients or students.

SECTION 07

Interpreting Your Results

What to look for first

Start with the shape of the frontier. If the curve bows significantly to the left, the two assets have a low (or negative) correlation and combining them produces a large diversification benefit. If the frontier is nearly straight, the assets are highly correlated and there is little benefit from mixing them beyond picking the better performer.

Reading the optimal weights

The stats strip below the chart button shows the Max Sharpe weights. For example, "Weight A: 68% · Weight B: 32%" means you should invest 68 cents of every rupee/dollar in Asset A and 32 cents in Asset B to achieve the best risk-adjusted return. If you are more conservative, look at the Min Variance weights instead.

When the frontier is nearly flat

If both assets have similar returns and high correlation, the frontier will be a short, nearly flat curve. In this case, the Max Sharpe and Min Variance portfolios will be close to each other, and the choice of weight matters less. This is a signal that the two assets are not good diversifiers of each other — consider replacing one with a less correlated alternative.

The risk-free rate matters

The Capital Market Line and the Sharpe Ratio both depend critically on the risk-free rate. If you set a low risk-free rate (say 2%) when the prevailing rate is 7%, the Sharpe ratios will look artificially high and the Max Sharpe portfolio will shift. Always use the current rate for the currency and market of your assets.

> **Important caveat — past performance**
>
> All calculations use three years of historical data. Historical return and volatility are inputs, not forecasts. The optimal weights the tool produces are optimal for the past three years. For forward-looking decisions, use the historical figures as a starting point and adjust with your own return forecasts. The tool supports manual override of all parameters.

SECTION 08

Saving Models & Exporting PDFs

Two productivity features — **Save Model** and **Export PDF** — are available to Trial and Premium users. Free users can still run the full optimisation and read every result on-screen; saving and exporting require an account.

FEATURE	FREE	TRIAL	PREMIUM
Plot Frontier	✓ Unlimited	✓ Unlimited	✓ Unlimited
Export PDF	✗ Not available	✓ Unlimited	✓ Unlimited
Save Model	✗ Not available	Up to 3 models	✓ Unlimited
Load Saved Model	✗ Not available	✓ All saved models	✓ All saved models

Exporting a PDF Report

Click the ⬇ **Export PDF** button (below the stats strip) after the frontier has been plotted. The tool generates a formatted A4 landscape PDF entirely in your browser — no data is sent to a server for PDF generation — and downloads it immediately.

PARAMETERS BAR

A dark bar at the top of the PDF lists every input used: Asset A and B tickers with their return and risk figures, the correlation coefficient, and the risk-free rate. This makes the report fully self-contained — a reader can reproduce the analysis without access to the tool.

CHART IMAGE

A high-resolution snapshot of the efficient frontier chart — scatter cloud, frontier curve, CML, ★ Max Sharpe marker, and ◆ Min Variance marker — is embedded in the PDF at full width.

RESULTS SUMMARY

Below the chart, a two-row summary banner shows the Max Sharpe Portfolio (weights in A and B, return, volatility, Sharpe ratio) and the Minimum Variance Portfolio (same metrics) — the take-away numbers you would present in a report or assignment.

HEADER & FOOTER

The Project Title, Name, Roll Number, and Date (if entered in the left panel) appear in the PDF header and footer. Filling these in before exporting produces a submission-ready document with your details and the FinMBA brand.

> Fill in the **Project Title, Name, Roll, and Date** fields before exporting. They appear in the PDF header and footer, making the report suitable for course submissions without any editing after download.

Saving and Loading Models

A **model** is a named snapshot of all your current inputs: both ticker symbols, their fetched return and volatility figures, the correlation, the number of random portfolios, the risk-free rate, and all project info fields. Saving a model lets you return to a previous analysis in any future session with a single click.

1 CLICK "SAVE MODEL"

The Save Model button appears below the Export PDF button once you are logged in with a Trial or Premium account. Click it after running the optimisation you want to keep.

2 ENTER A MODEL NAME (UP TO 10 CHARACTERS)

Choose a short, memorable name — for example RELHDFCB, NIFGOLD, or AAPL-MSFT. The name is used to identify the model in the dropdown. Names are unique per account; saving with an existing name prompts you to confirm overwrite.

3 LOAD IT ANY TIME FROM THE DROPDOWN

Select any saved model from the Load saved model dropdown. All inputs are restored instantly and the frontier is re-plotted automatically, so you can continue from exactly where you left off.

Trial — up to 3 saved models

During a Trial, you can save up to three distinct models. Overwriting an existing model (saving under the same name) does not consume an additional slot — it updates the existing record. When the three-model limit is reached, you must overwrite an existing model or upgrade to Premium to save more.

Premium — unlimited saves

Premium users can save as many models as needed with no cap. This is especially useful for tracking multiple portfolio pairs over time — for example, saving a monthly snapshot of the same two assets to see how the optimal weights shift as market conditions evolve.

What exactly is stored in a saved model?

Both ticker symbols · Annualised return and volatility for each asset · Correlation coefficient · Number of random portfolios · Risk-free rate · Project Title, Name, Roll Number, and Date. All values are stored as entered at the time of saving — the tool does not re-fetch live Yahoo Finance data when you load a model. This is intentional: it preserves the exact historical snapshot you analysed, so your results are reproducible.

SECTION 09

Glossary of Terms

A quick-reference table of every technical term used in the tool and this guide.

TERM	DEFINITION	IN THIS TOOL
Annual Return	The average yearly percentage gain or loss of an asset. Calculated here as average monthly return × 12 (arithmetic annualisation).	Auto-filled from Yahoo Finance. Shown in Ann. Return % field.
Annual Volatility	The annualised standard deviation of returns. Measures how widely returns fluctuate around their average. Higher = more uncertain / riskier.	Auto-filled. Shown in Ann. Volatility % field. X-axis of the chart.
Correlation (ρ)	A number between −1 and +1 that measures how two assets move together. +1 = perfectly in sync; −1 = perfectly opposite; 0 = independent. The lower the correlation, the greater the diversification benefit.	Auto-computed from aligned monthly returns. Shown on the slider.
Sharpe Ratio	(Expected Return − Risk-Free Rate) ÷ Volatility. Measures excess return earned per unit of risk. Popularised by William Sharpe (Nobel Prize 1990). Higher is better. Above 1.0 is generally good.	Encoded as dot colour in the chart (blue = low, red = high). Shown in the stats strip for the Max Sharpe portfolio.
Efficient Frontier	The curved boundary of all optimal	The solid black

	portfolios. For every level of risk, the frontier portfolio offers the maximum achievable return. No portfolio can exist above or to the left of this curve.	curve on the chart.
Opportunity Set	The full set of all possible portfolios — every weight combination from 0%/100% to 100%/0% — shown as the dot cloud. Portfolios inside the cloud are dominated (inefficient).	The coloured dot scatter on the chart.
Capital Market Line (CML)	A straight line from the risk-free rate through the Tangency Portfolio. Represents the best possible risk-return combinations when investors can also hold cash or borrow. The slope of the CML equals the maximum Sharpe Ratio.	The teal dashed straight line on the chart.
Tangency Portfolio	The portfolio at the point where the CML is tangent to the Efficient Frontier. It has the highest Sharpe Ratio of all portfolios on the frontier. Also called the Max Sharpe portfolio or Market Portfolio.	The ★ star marker on the chart.
Market Portfolio	In CAPM theory, the portfolio of all risky assets held in proportion to their market value. In practice, approximated by a broad index. In this two-asset context, it equals the Tangency Portfolio.	Same as the ★ star marker.
Minimum Variance Portfolio (MVP)	The portfolio with the lowest possible volatility. It is the leftmost point on the Efficient Frontier. Maximises risk reduction without any regard for return level.	The ◆ gold diamond on the chart.
Risk-Free Rate (Rf)	The return available from a zero-risk investment, typically a short-term government bond. In India, the 91-day T-Bill (~6.5-7%). In the US, the 3-month Treasury Bill (~5%). Acts as the y-intercept of the CML.	The Risk-free rate % input field. Default 4.5%.
Mean-Variance Optimisation	The Markowitz framework for finding efficient portfolios using only expected returns, variances, and covariances. Assumes investors are rational and care only about mean (return) and variance (risk).	The mathematical engine behind the entire tool.
Monte Carlo Simulation	Generating a large number of random outcomes to approximate a distribution. Here, random portfolio weights are generated to paint the full opportunity set visually.	The dot cloud. Controlled by the Random Portfolios input.
Portfolio Weight	The fraction of total capital invested in each asset. Weights sum to 100%. A weight	X-axis of the hover tooltip

	of 70% in Asset A means 30% is in Asset B.	shows Weight A / Weight B for each dot.
Two-Fund Separation	Theorem stating that every investor's optimal portfolio is a combination of the same risky portfolio (Tangency Portfolio) and the risk-free asset — differing only in the proportions based on risk appetite.	Embodied by the Capital Market Line.
Diversification Benefit	The reduction in portfolio risk achieved by combining imperfectly correlated assets. The portfolio's volatility is less than the weighted average of the individual volatilities whenever $\rho < 1$.	Visible as the leftward bow of the Efficient Frontier curve.
Standard Deviation (σ)	A statistical measure of how spread out values are around an average. In finance, the standard deviation of returns is the standard measure of risk (volatility).	X-axis (Ann. Volatility) and the Vol values in tooltips and the info banner.
Annualisation	Converting a monthly (or other period) figure to a yearly equivalent. Returns are annualised by multiplying by 12. Standard deviations are annualised by multiplying by $\sqrt{12}$, because variance scales linearly with time but standard deviation scales with the square root of time.	Applied automatically to all Yahoo Finance data before populating input fields.

← BACK TO MARKOWITZ FRONTIER